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VA, VAR and Power Factor – Fact or Fiction?

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Modern-day power measurement instruments offer a wide range of precise results, but do we really understand what they mean?

Volts, Amps, Watts – these are the real physical quantities. We can feel Volts (but don't try this!), detect the heat caused by Watts (be careful trying this one), and note the magnetic field caused by Amps. What about such things as VA, Power Factor, VAR, Crest Factor, THD, and K Factor? The simple answer is that these are not real! They have been defined by us humans as mere mathematical tools. So what's the problem you ask? Well, if we all used the same definitions and knew these definitions, no problem. But we humans aren't so simple. Many people do not recognize that these are not real quantities, or worse still, many have their own definitions (and everyone else is assumed wrong!) Let's look at some of these in more detail.

Single Phase VAR, VA and Power Factor

We'll deal with multi-phase power later, let's look at the "simple" case first.

VA and VAR

Many people take these results for granted, but they aren't so simple. The first thing to remember is that only V, A and W are actually measured. VA is simply the multiplication of the RMS measurements of Volts and Amps. Looking at power as a vector, with the Watts being the real part and VA as the scalar length, then VAR is the imaginary part. So $VAR^2 = VA^2 - W^2$. This is fine, but what about the polarity of VAR? What happens when more than one frequency is present?

The polarity of VAR should be dependent on whether the load is capacitive or inductive, generally people use positive VAR for a capacitive load (Xitron uses this definition), but not everyone follows this convention. So what happens when the VAR is "created" by distortion rather than phase shift? Well this leads to some "weird" effects. Take the example of a direct line rectification circuit with no PF correction. The current is drawn in a short little portion of time right at the peak of the voltage waveform. If this current is a little before the voltage peak, then VAR is positive (the current leads the voltage). If the current slightly lags the voltage peak, then VAR is negative. So if the current peak is right at the voltage peak, then the polarity of VAR switches between positive and negative due to noise, but the number stays the same! Does this matter? Well, if you're interested in long-term averages or accumulated totals it is! The



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long term total VAR (i.e. VAR-Hr) of such a system is measured as being close to zero, but the total Watts and VA are unaffected. This leads to the results conflicting, and the user is left scratching his (or her) head wondering what can be going on here!

Another anomaly, imagine a load consisting of a perfect one ohm resistor in series with a perfect infinite value capacitor. If we drive this load with a 1 volt RMS sine wave, then the Amps are 1, the Watts are 1, the VA is 1, and thus the VAR is 0. Just as we expected, right? But if we drive the load with 1 volt DC at the same time, what happens? Well, the RMS Volts are now 1.414V, the current is unchanged (the capacitor blocks the DC), and the Watts are unchanged. So, VA is now 1.414, but Watts are 1, so VAR must be 1. Let's see if we got that. Adding a DC signal created a reactive power! It's really just a matter of definition, remembering that VAR isn't a real thing. Don't expect it to produce real world results!

Power Factor

There are actually many definitions for this. Generally, $PF = W/VA$ is the one used, but $PF = VA \cdot \cos(\text{phase})$ is common, as is $PF = 1/\sqrt{1+THD^2}$. Aren't these the same? Not in the real world! The $\cos(\text{phase})$ equation only yields the "correct" result when no distortion or other frequencies are present. The THD related equation only yields the "correct" result when there is no phase shift present. These are both simplifications of the W/VA definition. Did we know that? A lot of people don't. A common customer service call at Xitron is from a person with a rectification circuit. The question is that since the current is exactly in phase with the voltage ("I looked at it with a 'scope and the current is right at the peak voltage so there's no phase shift") so the Power Factor "must" be 1! True, if you use the $\cos(\text{phase})$ simplified definition, but not if you use the full W/VA definition. PF should have polarity right? Correct, but not the one that many use. Many use the polarity of PF to indicate leading or lagging, this is not correct. The polarity of PF is the same as that of Watts (VA is a scalar, so has no polarity). It is the polarity of VAR that follows the phase polarity. Why is this error so commonplace? Well, in the "old" days, instruments couldn't display words like "lead" or "lag", and they didn't display VAR either. So, they indicated leading or lagging by means of a + or - symbol preceding the PF result (negative Watts are rather rare). This simple move to enable the user to see a leading or lagging indication has led to this popular misconception about the polarity of PF.

Crest Factor

What could be the problem here? $CF = \text{Peak}/\text{RMS}$, right? Well, there is a "simple" definition issue here, which many have not recognized. Should the Crest Factor include the effects of amplitude modulation or not? Why does it make a difference? Well, if there is significant amplitude modulation then the "long-term" peak is of the fullest part of the waveform, but the RMS value is of the complete waveform. Thus, the CF result is higher than if one looked at one or two cycles on an oscilloscope. Given this, there are two Crest Factors. Which one do you use?



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There is another issue at play here as well. Let's look again at our Crest Factor equation, $CF = \text{Peak}/\text{RMS}$. The Crest Factor of a waveform is defined as the ratio of the peak excursion from zero and the RMS value of the waveform. If so, then what the "peak" value truly is, and the "peak" value that one measures, may very well be different numbers depending on the frequency content of the "peak," and the bandwidth of the equipment measuring it. For example, if the signal of interest had a 10ns peak spike, and the bandwidth of the measurement instrument had a 1MHz bandwidth, you would never measure the true peak value. Along similar lines, in a digital sampling system such as most of today's test equipment, if the sampling rate of the instrument was 500ns (twice the bandwidth of the 1MHz system noted above so that Nyquist is happy), the chances of even catching that peak value are slim to non-existent!

THD and K Factor

Surely there can't be a problem with THD, can there? Well, do you measure the THD in absolute units, relative to the fundamental, relative to the total harmonics, or relative to the total signal? And should we include DC? When you read a power supply specification and the current THD is less than 50%, what does the manufacturer mean? Unfortunately, it's a pretty good bet that the definition yielding the lowest result is the one used!

Not that many people use K Factor, but it is common in certain areas of the power industry. However, few know its definition, or its limitations.

$K = \text{sum}(H^2 * A_h^2) / \text{sum}(A_h^2)$; where H = harmonic #, and A_h = amplitude at that harmonic.

Looks pretty simple, but the end result is often very dependent on how many harmonics have been used for the sum. Any K Factor result must include the number of harmonics used for the result to be useful to others, but it is rarely seen. We have seen many different definitions for the number of harmonics. There may be a standard, but many don't use it. Why is it so sensitive? Just imagine a square-wave current waveform (not that uncommon), the amplitude of each odd harmonic is proportional to the inverse of the harmonic number. The result is very dependent on the number of harmonics being summed, and the more distorted the waveform, the worse this gets.

Multi-Phase VA, VAR and PF

Now this is where the whole concept of these "physical" measurements just simply falls apart. There are several ways of combining the individual phase (or phase-to-phase) results to produce a set of total results, but these are all definitions by us humans, so are subject to interpretation. There are two ways to look at this, either we assume that only a single frequency is present (i.e. no distortion and no switch-mode generators), and that the phase loads are reasonably balanced, or we view the individual phase Watts as vectors. The two basic combination methods are;



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Wye (4-wire) Measurements

Either

$$\begin{aligned}\text{Total W} &= \text{sum(W)} \\ \text{Total VA} &= \text{sum(VA)} \\ \text{Total PF} &= \text{sum(W)} / \text{sum(VA)} \\ (\text{Total VAR})^2 &= (\text{Total VA})^2 - (\text{Total W})^2\end{aligned}$$

Or

$$\begin{aligned}\text{Total W} &= \text{sum(W)} \\ (\text{Total VA})^2 &= (\text{Total W})^2 + (\text{Total VAR})^2 \\ \text{Total PF} &= \text{sum(W)} / \text{sum(VA)} \\ \text{Total VAR} &= \text{sum(VAR)}\end{aligned}$$

Looks pretty similar don't they? However, they yield different results when distortion or switch-mode signals are present, or when loads are unbalanced. Remember, only the Watts results are "real".

Imagine a load where all three phases are resistively loaded, but one has a slightly "leading" PF and another has a slightly "lagging" PF. In this case the first definition will show a Total PF slightly less than one, while the second one yields a unity Total PF result (the VAR of opposite polarity cancel out each other, the end result is akin to an "average" PF). Which is correct? One could argue that one for a long time!

Delta (3-wire) Measurements

Either

$$\begin{aligned}\text{Total W} &= \text{sum(W)} \\ \text{Total VA} &= \text{sqrt}(3) \cdot (\text{sum(VA)}/2) \\ \text{Total PF} &= \text{sum(W)} / (\text{sqrt}(3) \cdot (\text{sum(VA)}/2)) \\ (\text{Total VAR})^2 &= (\text{Total VA})^2 - (\text{Total W})^2\end{aligned}$$

Or

$$\begin{aligned}\text{Total W} &= \text{sum(W)} \\ (\text{Total VA})^2 &= (\text{Total W})^2 + (\text{Total VAR})^2 \\ \text{Total PF} &= \text{sum(W)} / \text{sum(VA)} \\ \text{Total VAR} &= \text{sum(VAR)}\end{aligned}$$

Now we have some more noticeable differences. What's this square root of three? Well, if one assumes that the source is a 120° three-phase sine wave voltage, this is the result of the mathematical simplification. Why use this? Well, in the "old" days we couldn't measure Watts



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(or VAR), thus we needed a way to combine the two VA measurements (easily made with simple equipment) to produce an overall VA.

An interesting note, if one assumes that the neutral in the 4-wire case does not need to be exactly “in the middle”, then it could be coincident with one of the phases (then one of the phase VA and W results would be zero). The 4-wire case just became the 3-wire case. Only the second set of definitions yields the same equations for both 3-wire and 4-wire, so “by inspection”, the second method is independent of the assumption that the neutral is centrally located between the phases.

Conclusion

I’ve always been told that there should be a conclusion, but can there really be one? Since the various different definitions (or methods, whichever word you prefer) aren’t going away, we have to learn to deal with them. We must all recognize that these VA, VAR, etc., “measurements” are just numbers, not measurements at all. To interpret these, we must know the actual definition used to “measure” them, be that the mathematical method, the bandwidth or the time interval, they all affect the ability of the end-user to interpret the results.

Please note, Xitron Technologies Inc. produces a booklet containing all of the methods and mathematics used in the computation of the results given by its Power Analyzers. It is available free of charge from Xitron upon request, or may be downloaded from www.xitrontech.com.

Personal CV

Paul Withers has a B.Sc(hons) from Southampton University and has been Chief Engineer for Solartron/Schlumberger Digital Instruments, VP-Engineering for Valhalla Scientific and President/Chief Technical Officer for Xitron Technologies during his 30+ year career. He is also internationally published in peer reviewed medical research journals in the field of bio-impedance spectroscopy for *in vivo* human body fluid analysis. Paul can be reached at pwithers@xitrontech.com.